

Subtraction Games Solutions

$N(20; 1, 3, 5)$

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>

Let your opponent make the first move. Removing 1, 3 or 5 will put your opponent in an unsafe position.

$N(20; 1, 2, 5)$

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>

You should make the first move and remove 2 or 5 in order to be in a safe position.

$N(20; 1, 2, 6)$

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
<i>U</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>

You should make the first move and remove 6 in order to be in a safe position.

$N(20; 2, 3, 5, 7, 11, 13, 17) - \text{Primes}$

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
<i>S</i> <i>or</i> <i>D</i>	<i>S</i> <i>or</i> <i>D</i>	<i>S</i> <i>or</i> <i>D</i>	<i>U</i>	<i>S</i> <i>or</i> <i>D</i>	<i>S</i> <i>or</i> <i>D</i>	<i>S</i> <i>or</i> <i>D</i>	<i>U</i>	<i>S</i> <i>or</i> <i>D</i>	<i>U</i>	<i>S</i> <i>or</i> <i>D</i>	<i>S</i> <i>or</i> <i>D</i>	<i>S</i> <i>or</i> <i>D</i>	<i>U</i>	<i>S</i> <i>or</i> <i>D</i>	<i>U</i>	<i>S</i> <i>or</i> <i>D</i>	<i>U</i>	<i>U</i>	<i>D</i>	<i>S</i>

*In this game it is possible to have the game end in a draw. If there is one left, there is no way to finish the game. Some of the moves will lead to the game ending in a draw. The numbers marked *S* or *D* means that depending on what your opponent does the game could end in a draw. The strategy should be to avoid leaving a prime number at the end of your turn. There is no way to guarantee a win.*

$N(20; 1, 2, 4, 8, 16)$ – Powers of 2

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>U</i>	<i>S</i>

You should make the first move and remove 2 or 8 in order to be in a safe position.

 $N(20; 1, 3, 9)$ – Powers of 3

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>	<i>U</i>	<i>S</i>

You should let your opponent make the first move. Any amount removed will put your opponent in an unsafe position.