

# Subtraction Games

NAME \_\_\_\_\_

A *take-away game* or *subtraction game* is a game in which each player removes some objects from a pile. The most common game of this type is known as Nim, and variations are sometimes called *Nim games*. The rules for take-away games are very straightforward:

**Play:** Players alternate turns, each time removing some number of objects. Depending on which variation of the game is being played, the number of objects that may be removed varies.

**Object:** The player who removes the last object wins.

These games are denoted as  $N(X; a, b, c, \dots)$ , where  $X$  is the number of objects initially in the pile, and  $a, b, c, \dots$ , are the possible number of objects that can be removed.

So, for instance,  $N(50; 2, 3, 5)$  is a take-away game with 50 objects in the pile initially, and players can take 2, 3, or 5 objects on each turn.

(One variation to this notation is when all integers  $1, 2, 3, \dots, n$  may be taken, in which case the game is denoted as  $N(X; n)$ . That is, it is customary to not write out all possible moves if they correspond to the first  $n$  integers. As an example,  $N(50; 5)$  is the take-away game with 50 objects such that a player may remove 1, 2, 3, 4, or 5 objects on a turn.)

Analyze the following games. Each begins with a pile of 20 objects and allows the moves indicated.

**$N(20; 1, 3, 5)$**

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.

**$N(20; 1, 2, 5)$**

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.

**$N(20; 1, 2, 6)$** 

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.

 **$N(20; 2, 3, 5, 7, 11, 13, 17) - \text{Primes}$** 

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.

 **$N(20; 1, 2, 4, 8, 16) - \text{Powers of 2}$** 

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.

 **$N(20; 1, 3, 9) - \text{Powers of 3}$** 

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.